Hausaufgaben 17.2.2009 HW2A

Lösungen:

1 Bitte bestimmen Sie die Flächen unter den Funktionen

a)

$$\int_{\frac{2}{5}}^{\frac{4}{5}} \left| -\frac{3}{2}x^3 + \frac{19}{8}x^2 - \frac{5}{8}x \right| dx$$

L :

$$F(x) = \int -\frac{3}{2}x^3 + \frac{19}{8}x^2 - \frac{5}{8}x dx = -\frac{3}{8}x^4 + \frac{19}{24}x^3 - \frac{5}{16}x^2 + C$$

$$\int_{\frac{2}{5}}^{\frac{4}{5}} \left| -\frac{3}{2}x^3 + \frac{19}{8}x^2 - \frac{5}{8}x \right| dx = 0,0607$$

b)

$$\int_{\frac{-1}{2}}^{5} \left| \frac{5}{2}x^3 - \frac{8}{3}x^2 + \frac{2}{3}x \right| dx$$

L :

$$F(x) = \int \frac{5}{2}x^3 - \frac{8}{3}x^2 + \frac{2}{3}x dx = \frac{5}{8}x^4 - \frac{8}{9}x^3 + \frac{1}{3}x^2 + C$$

$$\int_{\frac{-1}{2}}^{5} \left| \frac{5}{2}x^3 - \frac{8}{3}x^2 + \frac{2}{3}x \right| dx = 288,0892$$

c)

$$\int_{\frac{2}{5}}^{1} \left| -\frac{5}{2}x^4 + 4x^3 + x^2 - 4x + \frac{3}{2} \right| dx$$

L :

$$F(x) = \int -\frac{5}{2}x^4 + 4x^3 + x^2 - 4x + \frac{3}{2} dx = -\frac{1}{2}x^5 + x^4 + \frac{1}{3}x^3 - 2x^2 + \frac{3}{2}x + C$$

$$\int_{\frac{2}{5}}^{1} \left| -\frac{5}{2}x^4 + 4x^3 + x^2 - 4x + \frac{3}{2} \right| dx = 0,0303$$

d)

$$\int_{\frac{1}{3}}^{\frac{5}{3}} \left| \frac{3}{4}x^4 + \frac{93}{80}x^3 - \frac{111}{80}x^2 - \frac{603}{320}x + \frac{27}{64} \right| dx$$

L :

$$F(x) = \int \frac{3}{4}x^4 + \frac{93}{80}x^3 - \frac{111}{80}x^2 - \frac{603}{320}x + \frac{27}{64} dx = \frac{3}{20}x^5 + \frac{93}{320}x^4 - \frac{37}{80}x^3 - \frac{603}{640}x^2 + \frac{27}{64}x + C$$

$$\int_{\frac{1}{3}}^{\frac{5}{3}} \left| \frac{3}{4}x^4 + \frac{93}{80}x^3 - \frac{111}{80}x^2 - \frac{603}{320}x + \frac{27}{64} \right| dx = 1,5009$$

e)

$$\int_{-\frac{2}{5}}^{3} -\frac{2}{3}x^4 + x^3 + \frac{13}{3}x^2 - 4x - \frac{20}{3}dx$$

L:

$$F(x) = \int -\frac{2}{3}x^4 + x^3 + \frac{13}{3}x^2 - 4x - \frac{20}{3}dx = -\frac{2}{15}x^5 + \frac{1}{4}x^4 + \frac{13}{9}x^3 - 2x^2 - \frac{20}{3}x + C$$

$$\int_{-\frac{2}{5}}^{3} -\frac{2}{3}x^4 + x^3 + \frac{13}{3}x^2 - 4x - \frac{20}{3}dx = -13{,}412$$

f)

$$\int_{-1}^{\frac{1}{2}} -2x^4 + \frac{59}{30}x^3 + \frac{98}{15}x^2 - \frac{142}{15}x + \frac{8}{3}dx$$

L:

$$F(x) = \int -2x^4 + \frac{59}{30}x^3 + \frac{98}{15}x^2 - \frac{142}{15}x + \frac{8}{3}dx = -\frac{2}{5}x^5 + \frac{59}{120}x^4 + \frac{98}{45}x^3 - \frac{71}{15}x^2 + \frac{8}{3}x + C$$

$$\int_{-1}^{\frac{1}{2}} -2x^4 + \frac{59}{30}x^3 + \frac{98}{15}x^2 - \frac{142}{15}x + \frac{8}{3}dx = 9{,}1266$$

2 Bitte bestimmen Sie die Fläche, die jeweils durch die beiden Funktionen eingeschlossen wird:

a)

$$f(x) = -\frac{1}{3}x^4 - \frac{17}{15}x^3 - \frac{37}{135}x^2 + \frac{19}{45}x + \frac{2}{15}$$

$$g(x) = \frac{2}{3}x^4 - \frac{71}{15}x^3 + \frac{452}{135}x^2 + \frac{13}{45}x - \frac{34}{45}$$

L:

Schnittpunkte:

$$S_1\left(-\frac{2}{5};\ -\frac{52}{3375}\right); S_2\left(1;\ -\frac{32}{27}\right);\ S_3\left(\frac{4}{3};\ -\frac{286}{81}\right);\ S_4\left(\frac{5}{3};\ -\frac{3136}{405}\right);$$

$$H(x) = \int x^4 - \frac{18}{5}x^3 + \frac{163}{45}x^2 - \frac{2}{15}x - \frac{8}{9}dx = \frac{1}{5}x^5 - \frac{9}{10}x^4 + \frac{163}{135}x^3 - \frac{1}{15}x^2 - \frac{8}{9}x + C$$

$$\int_{-\frac{2}{5}}^{\frac{5}{3}} \left| x^4 - \frac{18}{5}x^3 + \frac{163}{45}x^2 - \frac{2}{15}x - \frac{8}{9} \right| dx = \left| \frac{1}{5}x^5 - \frac{9}{10}x^4 + \frac{163}{135}x^3 - \frac{1}{15}x^2 - \frac{8}{9}x \right|_{-\frac{2}{5}}^{\frac{5}{3}} = 0{,}7014$$

b)

$$f(x) = 1;\quad g(x) = \frac{5}{4}x^2 + \frac{5}{4}x - \frac{3}{2}$$

L:

Schnittpunkte:

$S_1(-2;\ 1)$;

$S_2(1;\ 1)$;

$$H(x) = \int \frac{5}{4}x^2 + \frac{5}{4}x - \frac{5}{2}dx = \frac{5}{12}x^3 + \frac{5}{8}x^2 - \frac{5}{2}x + C$$

$$\int_{-2}^{1} \left| \frac{5}{4}x^2 + \frac{5}{4}x - \frac{5}{2} \right| dx = \left| \frac{5}{12}x^3 + \frac{5}{8}x^2 - \frac{5}{2}x \right|_{-2}^{1} = 5{,}625$$

c)

$$f(x) = -2 \quad ; g(x) = \frac{1}{4}x^3 + \frac{9}{80}x^2 - \frac{7}{40}x - \frac{163}{80}$$

L :

Schnittpunkte :

$$S_1(-1;\ -2);\ S_2\left(-\frac{1}{5};\ -2\right);\ S_3\left(\frac{3}{4};\ -2\right);$$

$$H(x) = \int \frac{1}{4}x^3 + \frac{9}{80}x^2 - \frac{7}{40}x - \frac{3}{80}dx = \frac{1}{16}x^4 + \frac{3}{80}x^3 - \frac{7}{80}x^2 - \frac{3}{80}x + C$$

$$\int_{-1}^{\frac{3}{4}} \left| \frac{1}{4}x^3 + \frac{9}{80}x^2 - \frac{7}{40}x - \frac{3}{80} \right| dx = \left| \frac{1}{16}x^4 + \frac{3}{80}x^3 - \frac{7}{80}x^2 - \frac{3}{80}x \right|_{-1}^{\frac{3}{4}} = 0{,}0743$$

3 Für ein Polynom gelten die folgenden Bedingungen. Bestimmen Sie die Funktionsgleichungen.

A)
- Grad 3
- an der Nullstelle 2 die Steigung 0
- schneidet die y-Achse bei 0,8 mit der Steigung 0

L:
f (2) = 0 ;
f' (2) = 0
f (0) = 0,8 ;
f' (0) = 0

8a + 4b + 2c + d = 0
12a + 4b + c = 0
d = 0,8
c = 0

a = 0,2 ; b = -0,6 ; c = 0 ; d = 0,8 ;

f (x) = 0,2x^3 - 0,6x^2 + 0,8

B)
- Grad 4
- Extremwert am Punkt (-0,2; 2)
- am Punkt (0; -2) die Steigung -0,6
- Nullstelle bei -0,4

L:
f (-0,2) = 2 ;
f' (-0,2) = 0
f (0) = -2 ;
f' (0) = -0,6
f (-0,4) = 0

0,0016a - 0,008b + 0,04c - 0,2d + e = 2
- 0,032a + 0,12b - 0,4c + d = 0
e = -2
d = -0,6
0,0256a - 0,064b + 0,16c - 0,4d + e = 0

a = 2775 ; b = 2095 ; c = 405 ; d = -0,6 ; e = -2 ;

f (x) = 2775x^4 + 2095x^3 + 405x^2 - 0,6x - 2

	C) - Grad 4 - symmetrisch - am Wendepunkt (1; 1) die Steigung -2,5 L: f (1) = 1 ; f' (1) = -2,5 ; f'' (1) = 0 d = 0 b = 0 a + b + c + d + e = 1 4a + 3b + 2c + d = -2,5 12a + 6b + 2c = 0 a = 0,3125 ; b = 0 ; c = -1,875 ; d = 0 ; e = 2,5625 ; **f (x) = 0,3125x^4 - 1,875x^2 + 2,5625**